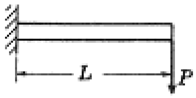
From: www.wwa.org.za - September 2007

Lathe Work holding – an analysis of forces

©Trevor Pope (tpope AT iafrica.com) – Sep 2007

Have you ever wondered why a long work-piece held at one end in the chuck on your lathe deflects so much when you try to cut it with a tool? If you studied mechanical engineering, you will have come across the models to predict this deflection. It may help your understanding to review some of the theory so you can understand why a longer work piece deflects so much more than a shorter one.

The diagram shows a beam, length L, solidly attached at one end with a point load (P) at the other. In our case, assume the work piece is firmly held in a chuck on the left and force exerted by the cutting tool is acting on the opposite end, on the right, approximating a point load.



The work piece is cylindrical, so paging through my old Applied Mechanics textbook formulae, I found one that predicts the deflection δ, for the above geometry:

$$\delta = \frac{64PL^3}{3E\pi D^4}$$

Where: P is a point load such as from a lathe tool
L is the length of the work piece from the chuck
E is the modulus of elasticity (how strong the material is)
D is the diameter of the work piece.

Absolute values are not important. This equation is to help you understand the effects of changing different values.

Logically, if you increase the load P, the deflection δ increases proportionally.

Also if you increase the length, the equation tells us that the deflection increases, but to the third power of the length. So if you double the length, the deflection increases eight times! (2^3) This explains why you can only take very light cuts on the tail end of a long work piece that is supported only on the headstock end. Alternatively, you need to add another support, such as bringing up the tail stock, or using a steady rest.

The stronger the wood, the larger is E, so the equation predicts less deflection, as one would expect.

Another insight is the effect of the diameter of the work piece. This is also logical – if you increase the diameter, the work piece becomes stiffer or if you reduce it, it deflects much more. The equation tells us that D is to the inverse fourth power! This means that if you double the diameter, the deflection goes down by a factor of $1/ 2^4 = 1/ 16$. The work piece becomes much stiffer. Conversely, if you halve the diameter, the deflection increases by a factor of 16!

This change in stiffness may help to explain why when making a goblet, the sequence of cuts is most important. When turning a goblet from a cylinder of wood, imagine the work piece is held in the chuck at the end where the base will be. The correct way to make a goblet is to hollow out and finish the cup on the tail stock end before thinning down the stem. You start by shaping the outside of the cup and hollowing out the inside to a uniform wall thickness. Then, you can turn down the stem and finally part off the base. When the stem is cut down, it could have a diameter one tenth (or less) of that of the cup, so the stiffness of the stem decreases by a factor of 1 / 10000 ($1 / 10^4$). This explains why once the wood is cut down to the thickness of the stem, there will be so much deflection from any pressure on the cup, that it will be impossible to make any cuts on the cup. Only light sanding is possible.

The maths helps to explain why the sequence of removing material when making an object is sometimes so important.

(If you wish to make some real calculations, the Encyclopaedia of Wood published by the American Forest Products Laboratory and Sterling 1987 gives tables of E. This depends on the direction of the grain, but working values of 1 to 2 million psi are a start.)